Strategic Asset Allocation in a Multi-Factor World
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Strategic asset allocation is arguably one of the most important, yet least advanced, aspects of investing. The Investment Strategy Group (ISG) in the Goldman Sachs Investment Management Division has developed a new approach to strategic asset allocation, which leverages the idea that long-term investment returns derive from multiple distinct sources called "return-generating factors." This *multi-factor approach* is designed to help investors better understand the key sources of long-term return across asset classes and to increase the precision of long-term risk and return estimates. It also provides investors with a new way to think about portfolio diversification, allowing them to focus not only on diversification across asset classes but also on diversification across the underlying sources of return.

The factor-based risk and return estimates are used in a *robust portfolio optimization process* that the ISG has designed to address each client's individual investment goals and preferences. The factor model is also leveraged in portfolio stress tests that help the ISG better capture downside risks, including tail events and elevated correlations between asset classes at times of crises.

Previously, the ISG's asset allocation approach was based on the single-factor model of Black and Litterman and the assumption of Normal distribution for asset returns. These models were supplemented by qualitative assumptions in different parts of the investment process. For example, adjustments designed to improve the estimates of long-term returns were applied to a number of asset classes, and additional risk metrics were relied upon to approximate tail risks not captured by the Normal distribution.

While we believe the new multi-factor framework addresses many shortcomings of traditional asset allocation approaches, quantitative models are only part of the ISG's investment framework. To help mitigate model risks and to understand individual investors' specific goals and softer preferences, qualitative judgment and investment expertise remain important inputs to the asset allocation process.

Investment Strategy Group. The Investment Strategy Group (ISG) is focused on asset allocation strategy formation and market analysis for Private Wealth Management. Any information that references ISG, including their model portfolios, represents the views of ISG, is not research and is not a product of Global Investment Research. If shown, ISG Model Portfolios are provided for illustrative purposes only. Your actual asset allocation may look significantly different based on your particular circumstances and risk tolerance

Description of Factor Model and Robust Optimization. We use our proprietary factor model and robust optimization process to construct a long-term asset allocation that has the potential to provide clients with the greatest long-term expected return given your investment goals and risk tolerance.

Our approach begins by establishing the risk and return characteristics for each asset class that could potentially be included in your portfolio. We use representative indices for asset classes to arrive at all estimates. We have identified several factors that we believe drive long-term risk and return, including systematic equity risk, inflation and interest rate risk, and market-wide liquidity risk. By estimating each factor's contribution to the risk and return of each asset class, we establish three key attributes:

- *Estimated Mean Return* is our estimate of the average annual return of the asset class over long periods of time. Each asset class' Estimated Mean Return is the sum of two components: (1) the theoretical rate of return on a riskless investment, or the "*Risk-Free Rate,"* and (2) the estimated long-term return on an annual basis in excess of the Risk-Free Rate, or the "Risk Premium"
- *Estimated Ranges of Risk Premia*. We express the Risk Premium of each asset class as a specified percentage plus or minus an estimated range. For example, U.S. Investment Grade Bonds have a Risk Premium of 1.7% +/- 0.8%. The estimated range for each asset class reflects the level of certainty we have regarding our Risk Premium estimate. A larger range reflects a lower level of certainty.
- *Long-term Risk*. We use two primary measures to quantify the risk of each asset class: volatility and correlation. Volatility measures the possible fluctuation in the return of each asset class. Correlations measure the linear relationships of each asset class' return with the returns of other asset classes. Volatilities of, and correlations across, asset classes included in a portfolio are used together to determine the overall risk of a portfolio.

We run our robust optimization process using the investment goals and risk tolerance you have shared with your Private Wealth Management team and the asset class attributes described above. The process considers all potential asset allocation alternatives before arriving at the allocation that offers the greatest expected return with the greatest level of certainty given your investment goals and risk tolerance. The output of the optimization process is the target strategic asset allocation that we share with you. The results shown reflect the reinvestment of dividends and other earnings but do not reflect advisory fees, transaction costs, and other expenses a client would have paid, which would reduce return.

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Strategic asset allocation is arguably the most important, yet one of the least advanced aspects of investing. Well-known studies have shown that asset allocation policy benchmarks explain on average 90% of the variability and as much as 100% of the levels of portfolio returns over time.[1] Yet strategic asset allocation tools have evolved little since Black and Litterman introduced their equilibrium approach in the early 1990s.[2] This lack of development is particularly stark when contrasted with the substantial innovations in other areas of finance, such as securitization and statistical arbitrage.

Against this backdrop, we developed a new approach to strategic asset allocation. It leverages the idea that long-term investment returns derive from multiple distinct sources that we call *return-generating factors*.[3] Our approach is comprised of four key innovations designed to benefit investors.

First, we built a *multi-factor model* that identifies important sources of return in today's complex investment universe, letting us substantially increase the precision of expected return estimates across asset classes. It also allows investors to shift their focus from diversification across asset classes to diversification across risk premia.

Second, we developed a *robust portfolio optimization* methodology that seeks to explicitly account for the uncertainties inherent in the estimates of expected returns, delivering well-diversified portfolios with superior risk/return characteristics.

Third, we designed *factor-based risk analytics* to better capture the real-world characteristics of asset returns, such as fat tails and increased correlations at times of crises, allowing us to more accurately model portfolios' downside risks.

Finally, we developed a *factor-based simulation* technique to account for the impact of different economic conditions, such as low interest rates, on future portfolio returns, resulting in more precise forward-looking projections.

In short, we expect our new approach to help investors make better investment decisions that more accurately reflect their individual investment goals and constraints. It will also let investors better understand potential portfolio behavior in a broad range of market environments, including periods such as the crisis of 2008–2009, when traditional diversification methods proved so disappointing.

A FACTOR-BASED APPROACH TO STRATEGIC ASSET ALLOCATION

Whatever the investor's specific objective, the process of strategic asset allocation begins by understanding the investment opportunities the global markets offer. One must gain an understanding of the expected returns and risks of the available asset classes, ranging from public equities and bonds to alternative investments and private assets. Traditional approaches attempt to tackle this challenge via the capital asset pricing model (CAPM) of Sharpe [1964], Lintner [1965], and Mossin [1966], and its various global extensions, including Black and Litterman's model [1992], which to date remains the industry standard among practitioners. These approaches are based on single-factor models in which the only factor is the market's excess return, typically represented by a market capitalization-weighted portfolio of stocks or stocks and bonds. The approaches imply that assets with higher market exposures, or *betas*, must be riskier and therefore command higher expected returns.

However, since the CAPM's introduction, ample empirical and theoretical evidence has surfaced to suggest that the world is substantially more complex than single-factor models can allow. Market participants worry about many risks, above and beyond market risk, and are willing to pay handsome rewards to investors willing to bear those risks. The premium associated with market risk is not the only dependable source of return, as long-term returns also derive from a number of other global risks.

Based on this idea, multi-factor models were developed, originating in the work of Merton [1969, 1973] and Ross [1976], and later refined by Fama and French [1992], among others. In these models, every factor, such as size or value, reflects a distinct risk premium. Practitioners have also developed factor models for risk, which focus solely on explaining assets' volatility and co-movement. Risk models typically include a larger number of factors, most of which do not carry risk premia.

To the best of our knowledge, our factor model represents the first broad-based application of multi-factor models to strategic asset allocation: estimating expected returns and risk *across* asset classes. In contrast, previous applications of multi-factor models have focused primarily on estimating expected returns and risk *within* asset classes. Hedge funds and quantitative equity funds have used factor models as a basis for security selection in systematic trading strategies for decades. For example, these funds might capture the value premium by forming portfolios that are long value stocks and short growth stocks. Equity managers have also used multi-factor risk models since the 1970s. Today, factor-based risk models are increasingly applied to other asset classes also.[4]

Our factor model for expected returns consists of six factors (see Exhibit 1), which represent sources of long-term return for a global investor.[5] We emphasize four key model characteristics. First, each factor reflects a distinct risk premium that is largely independent of the others. This feature lets us think about portfolio diversification in a new way, which we will discuss later.

Second, each risk premium has a clear economic rationale, as shown by some market participants' willingness to pay a premium to offload the risk.

Third, the reward associated with each factor reflects compensation for systematic risk in the cross-section of expected returns. By implication, assets that have higher exposures to our factors are expected to earn higher returns.

Finally, each risk premium is investable. Factor returns can be achieved via long and short positions in liquid, tradable assets.[6]

To illustrate how the multi-factor model helps us better understand sources of return, consider an example: macro/tactical hedge funds. These strategies have low exposures (betas) to market risk, so traditional single-factor approaches shed little light on their risk premium: the CAPM predicts a total risk premium of only 0.2%, leaving a substantial portion of the historical average excess return unexplained. In contrast, our factor model assigns a total risk premium of 3% to the average macro/tactical strategy.

A similar result applies to the rest of our strategic investment universe. Relative to the CAPM, our multi-factor model more than doubles the average estimation precision. Importantly, our approach helps reduce the need for qualitative adjustments in long-term return estimates for diversified portfolios of hedge fund strategies or other alternative investments.

EXHIBIT 1

Six-Factor Model for Estimating Expected Returns

RISK PREMIUM	REWARDS INVESTORS FOR…
Equity	Market risk
Term	Inflation and interest rate risk
Funding	Risk in short-term credit conditions
Liquidity	Risk in market-wide liquidity conditions
FX	Systematic exchange rate risk
EM	Risks specific to emerging markets

Since each asset class can be represented as a combination of our six factors, and each factor represents a distinct source of long-term return, the factor approach provides us with a new way to understand the diversification benefits of different asset classes. To illustrate this, Exhibit 2 displays examples of risk premium profiles for three asset classes. Viewing the different asset classes through the six-dimensional lens of our factor model highlights the diversity of our investment universe. Each asset class has a unique risk premium profile, a distinct identity that is completely neglected by traditional approaches that rely solely on the equity premium. It seems clear that asset class names are merely labels for different sets of underlying factor exposures that ultimately drive their returns. Understanding these return drivers provides important insights for portfolio construction.

An ideal portfolio consists of many independent return-generating factors. Since our factors reflect sources of long-term return that are largely uncorrelated, they let us think about portfolio diversification in a new way. Rather than just focusing on diversification across asset classes, a long-term portfolio should derive its returns from a balanced profile of factor risk premia.

This is important because not all factors generate returns all the time. Over the past decade, for example, the equity premium has fallen significantly short of its long-term average. Under such circumstances, a well-diversified portfolio should continue harvesting returns from other risk premia. Using our factor model, we can identify each portfolio's unique risk premium profile, which helps us understand the degree of diversification at a deeper level.

Exhibit 3 provides an example for two hypothetical portfolios. From an asset allocation perspective, the portfolio on the left appears more diversified than the portfolio on the right. However, by inspecting the risk premia that underlie the return-generating power of each portfolio, it is clear that the portfolio on the right is substantially more diversified.

Understanding the composition of the risk premia that drive portfolio returns can also be helpful in medium-term asset allocation decisions. For example, in low interest-rate environments, investors may fear the impact of rising interest rates on their portfolios over the medium term. Historical return data for asset classes such as high-yield bonds, hedge funds, and many emerging-market securities rarely extend further than a couple of decades, but our factor returns do. That lets us derive more reliable estimates of expected asset class returns in different economic environments.

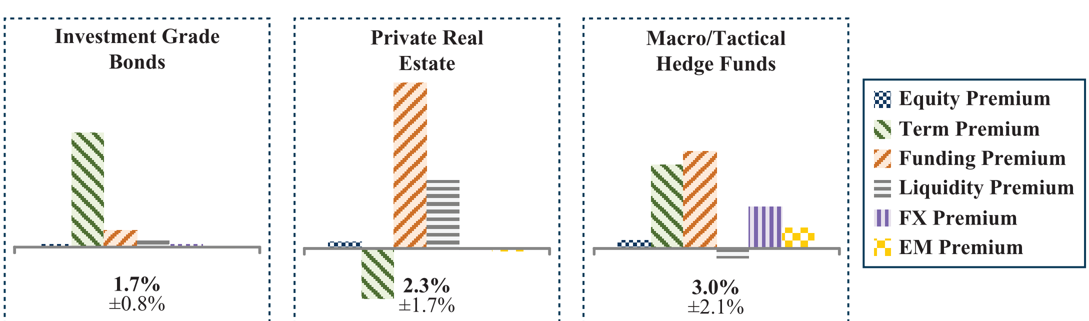
For example, our factor model estimates that all hedge fund strategies, with the exception of macro/tactical trading, should perform better in a rising interest-rate environment.[7] The reason for macro/tactical hedge funds' potential underperformance is their significant exposure to the term factor (see Exhibit 2), whose premium is estimated to turn negative when interest rates are rising.

ROBUST OPTIMIZATION IN STRATEGIC ASSET ALLOCATION

Having established a thorough understanding of the investment universe, the goal of optimization is to find the asset allocation that best meets the investor's

EXHIBIT 2

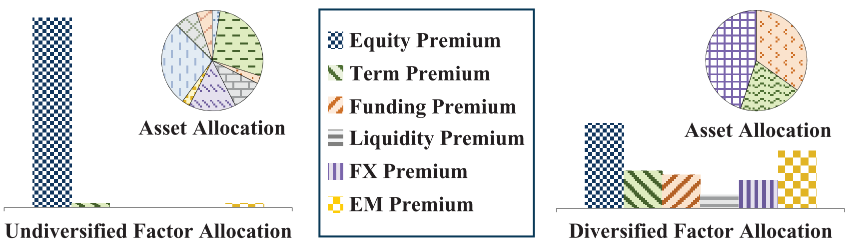
Examples of Risk Premium Profiles for Selected Asset Classes



Note: The total estimated risk premium and the associated standard error are displayed below each profile.

EXHIBIT 3

Diversification from a New Perspective



specific investment goals. To do this, most approaches formulate the investor's objective in terms of the portfolio's expected return and variance, building on the mean-variance framework of Markowitz [1952]. However, in the absence of restrictive constraints, direct application of mean-variance optimization generally results in undiversified portfolios that exhibit poor realized performance.

Many approaches have tried to address this well-known shortcoming,[8] including the Bayesian solution of Black and Litterman [1992], the re-sampling method of Michaud [1998], the risk parity approach favored by many practitioners (e.g., Asness et al. [2012]), and a number of robust optimization algorithms (e.g., Scherer [2007]). Of these approaches, we favor robust optimization for its analytical rigor and ability to accommodate a broad range of different investor objectives.

Our new robust optimization process leverages an important statistical fact, one ignored by traditional mean-variance frameworks. Even the best estimates of expected returns carry substantial uncertainties, also known as standard errors. That is, estimates of long-term asset returns today are likely to differ materially from the average returns realized in the future. Moreover, differences in data quality and availability imply that the risk/return characteristics of some asset classes are estimated with a higher precision than others.

It turns out that mean-variance optimization is very sensitive to such errors. Small changes in the input parameters can cause large changes in the composition of the optimal portfolio, often resulting in corner solutions. Errors in expected return estimates are particularly important. For a moderate-risk portfolio, Chopra and Ziemba [1993] report that the error sensitivity in expected return estimates is approximately one order of magnitude greater than the error sensitivity in variances.

Our robust optimization technique seeks to address this shortcoming by explicitly accounting for standard errors of expected returns in portfolio construction. An example of the resulting allocation is displayed on the right-hand panel of Exhibit 4, which stands in stark contrast to the undiversified allocation generated by traditional mean-variance optimization on the left. Note that each allocation is the result of an unconstrained optimization. Robust optimization lets us construct portfolios with fewer subjective inputs.

In casual parlance, the word "robust" is often misused as a synonym for "rigorous" in portraying methodological soundness. We use "robust" to reflect our optimizer's ability to account for parameter uncertainty. Mathematically, our formulation for robustness resembles that of Scherer [2007], Meucci [2007], and Ceria and Stubbs [2006], who extend traditional mean-variance optimization to incorporate uncertainty in expected returns. Recall that mean-variance optimization finds portfolio weights that solve:

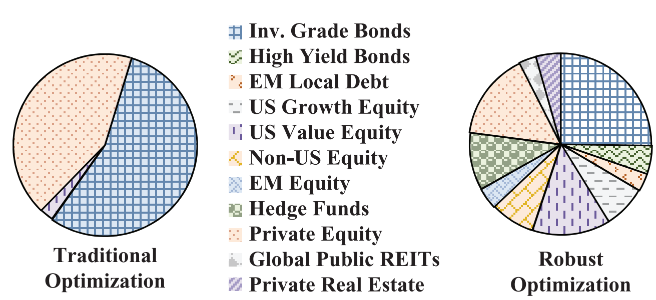
$$\begin{aligned} &\max_w E[w'R] \\ &\quad s.t. \quad Var[w'R] \le \sigma^2_{\text{Target}}; \quad \sum w \le 1 \end{aligned} \tag{1}$$

where $E[w'R]$ is the expected portfolio return, $Var[w'R]$ is the portfolio variance, and σ^2_{Target} is the target portfolio variance. Robust optimization extends this by solving:

$$\begin{aligned} &\max_w \{\min_{\tilde{R}\in U} E[w'\tilde{R}]\} \\ &\quad s.t. \quad Var[w'R] \le \sigma^2_{\text{Target}}; \quad \sum w \le 1 \end{aligned} \tag{2}$$

EXHIBIT 4

Robust Optimization Acknowledges that Expected Returns are Uncertain



where U represents the uncertainty set. Intuitively, our robust optimization process seeks to maximize the expected portfolio return for those unfortunate realizations of the world where our estimates of expected returns deviate the most from their "true" values.

To see how our robust optimizer copes with errors in expected return estimates, we conduct the following test. We first use our factor model to estimate expected returns and risk for 15 strategic asset classes, using data until December 2000. We then generate 1,000 perturbations of these expected returns by adding independent random errors drawn from normal distributions with zero means and volatilities that match the expected return estimates' standard errors.

We then feed these perturbed expected returns and the asset covariance matrix into our robust optimizer Equation (2) and the traditional mean-variance optimizer Equation (1) to arrive at respective portfolio weights for each of the 1,000 perturbations. We evaluate these portfolios' performances between 2001 and 2010. Exhibit 5 displays the results. One can see that the cumulative returns generated by our robust optimization process are less dispersed than those generated by traditional mean-variance optimization, both along the way (left panel) and at the end of the ten-year horizon (right panel).

Ceria and Stubbs [2006] provide a different test for the performance of portfolios constructed using this formulation of robust optimization. Rather than perturbing the expected returns, they simulate thousands of hypothetical market outcomes and evaluate the performance of robust optimal allocations relative to mean-variance optimal allocations.

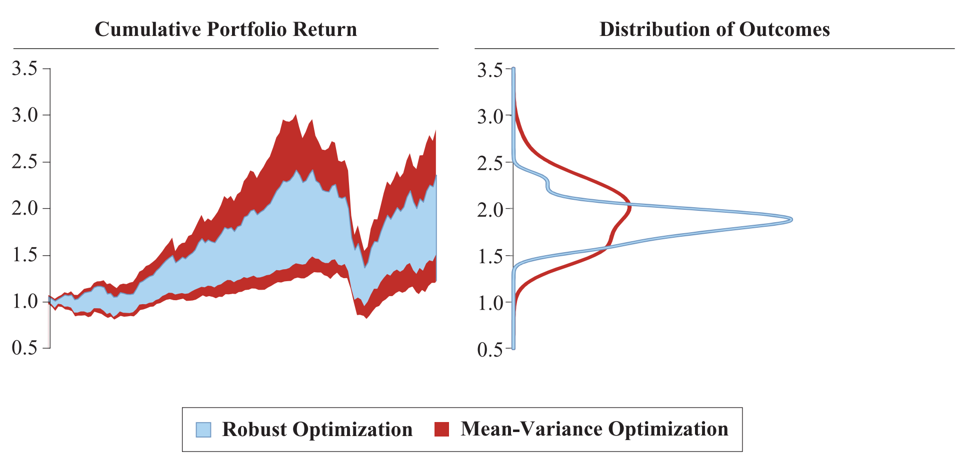
Their results demonstrate that robust optimization delivers greater average returns for the same level of risk, generating a higher *realized* efficient frontier. The expected returns "promised" ex ante by mean-variance portfolios tend to be higher than average portfolio returns realized ex post. Mean-variance optimization tends to over-promise but under-deliver. Ceria and Stubb's analysis shows that the ex ante expected returns of robust optimal portfolios are much closer to their ex post realizations. This feature, which stems from the greater diversification afforded by robust optimal portfolios, lets investors have higher confidence in their anticipated portfolio returns.

Robust Optimization versus Risk Parity

Among the approaches developed to address the high sensitivity of mean-variance optimization to errors in expected returns, risk parity may have received the most publicity. The solution that risk parity provides involves ignoring expected returns altogether and forming portfolios in which each asset class contributes the same amount of risk. As a result, typical risk parity portfolios have high allocations to bonds and other low-volatility asset classes, and rely on leverage to increase portfolio risk to the level the investor desires.[9]

Exhibit 5

Comparing Hypothetical Robust Optimal Portfolios and Mean-Variance Optimal Portfolios from 2001 to 2010



In our view, completely ignoring different asset classes' expected returns leads to suboptimal allocations. This is because not all risks receive a reward. Some risks are idiosyncratic and can be diversified away. Moreover, not all rewards are equal. The premium associated with a unit of liquidity risk, for example, may differ significantly from the premium associated with a unit of exchange rate risk. Our factor model for expected returns is a valuable tool for identifying and pricing the risks for which investors are rewarded.

Our robust optimization platform provides an effective way to use this information to help account for the substantial uncertainties that remain with the expected return estimates. After allowing for leverage, our robust optimal portfolios are readily comparable to risk parity portfolios. Exhibit 6 provides an example for a typical strategic investment universe of 21 asset classes. Our leveraged, robust optimal portfolio generates a higher expected return for the same level of risk, which can be attributed to greater diversification across risk premia. This result highlights the importance of optimally combining the different sources of return that drive long-term portfolio performance.

OTHER APPLICATIONS OF FACTOR MODELS IN STRATEGIC ASSET ALLOCATION

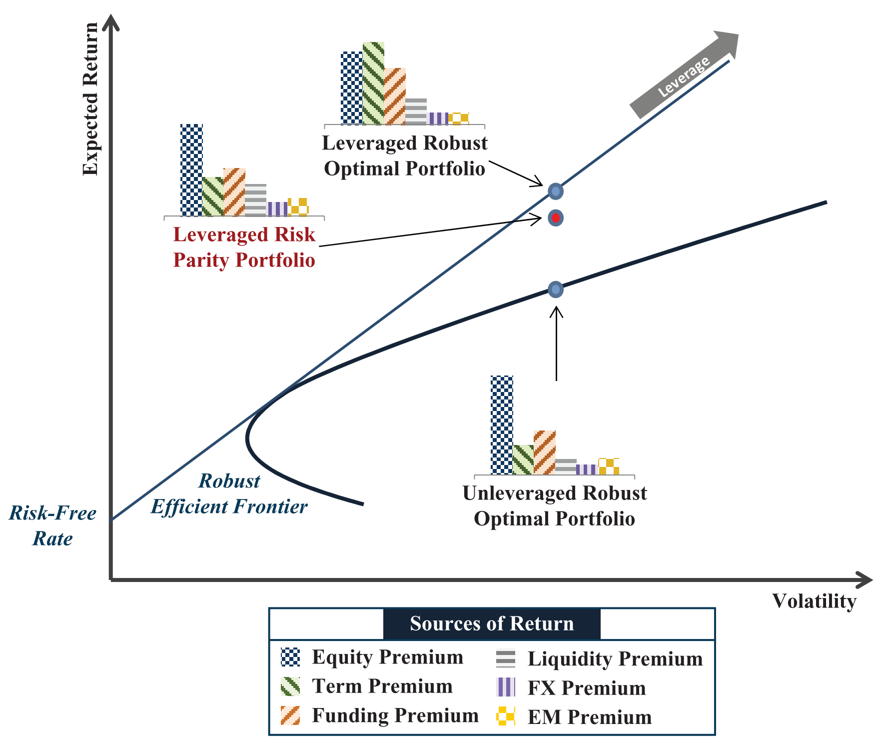
Factor-Based Risk Analytics

Understanding portfolio behavior in certain market environments, including distressed scenarios, is an integral part of the portfolio construction process. Here our factor-based methodology provides another material edge over traditional approaches.

Traditional approaches tend to assume that asset returns are normally distributed and that consecutive returns are independent of one another. These assumptions imply that the likelihood of events such as the financial crisis of 2008–2009 is practically zero. In reality, such *tail events* occur more frequently. They are often associated with sequences of bad returns, clusters of high volatility, and increased correlations among risky assets. Since the characteristics of our factors' distribu-

E X H I B I T 6

Robust Factor-Based Approach Generates "Smarter" Portfolios—An Example for a Universe of 21 Asset Classes



tions are passed through to modeled assets, our factor-based risk analytics and stress tests help capture these real features of portfolio returns. As a result, we are better equipped to measure the true downside risks investors face.

The factor approach also helps us estimate a given portfolio's potential performance during historical stress episodes, such as the 1973–1974 oil embargo or high U.S. inflation between 1978 and 1980. Historical return data for many asset classes is not available over these periods, making this an important contribution.

Factor-Based Portfolio Projections

Anticipating the effect of planned spending, inflows, inflation, and taxes on the distribution of future portfolio values is another crucial part of the strategic asset allocation process, especially for endowments and foundations with spending requirements. Our factor-based portfolio projections address two key shortcomings in the Monte Carlo simulation techniques that have traditionally been used to tackle this problem.

First, factors allow us to eschew the assumption of independent, normally distributed returns in favor of more realistic factor-based distributions. Second, our wealth projections can account for the prevailing economic environment's effect on projected portfolio performance. For example, starting in a low-interest rate environment, total portfolio returns projected over a decade will be affected not only by the convergence of the risk-free rate to its long-term average, but also by the adjustment of risk premia. Portfolio projections that leverage our factor model capture both features, resulting in a better understanding of the broad range of potential wealth outcomes.

CONCLUSION

Despite the importance to long-term investors, most strategic asset allocation decisions continue to be made based on techniques developed in the 1950s and 1960s. These tools have a number of shortcomings that limit their value to the investment process. We have addressed many of these shortcomings in our robust factor-based framework, which can be applied to important aspects of the asset allocation process, allowing investors to make better investment decisions that are more suited to their individual objectives and constraints.

Although we believe we have pushed the scientific envelope of strategic asset allocation techniques, the holistic process of asset allocation remains a combination of art and science. We have addressed a number of key shortcomings in the traditional science of asset allocation, substantially reducing the need to use art as a way to remedy these pitfalls. Still, art remains an important component in understanding individual investors' specific goals and softer preferences. Qualitative judgment, used to intuitively validate the investment recommendations suggested by the science, continues its importance.

ENDNOTES

[1]Ibbotson and Kaplan [2000] provide an excellent overview of this literature.

[2]See Black and Litterman [1992].

[3]We will describe these factors later.

[4]See for example Shepard [2011].

[5]Many of the return-generating factors listed in Exhibit 1 build on existing asset-pricing literature. For example, the funding premium is studied in Adrian, Etula, and Muir [2012]; the liquidity premium is investigated in Pástor and Stambaugh [2000]; the exchange-rate premium is studied in Lustig, Roussanov, and Verdelhan [2011]. Our risk model supplements the six return-generating factors with other factors that let us better capture long-term co-variances between assets. As discussed above, these additional risk factors do not carry risk premia.

[6]In recent years, a number of authors have investigated the idea of investing directly in factors (see, for example, Bender, Briand, Nielsen, and Stefek [2010], and Ilmanen and Kizer [2012]). For investors comfortable with both long and short positions in risky securities, portfolios of risk premia may provide ideal, highly tailored risk/return profiles.

[7]Interestingly, this is also what we have observed in the limited historical data on hedge funds.

[8]See Fabozzi et al. [2007] for an overview.

[9]Aggressive bond allocations are the main reason for the strong performance of risk parity portfolios in the decreasing-yield environment of the past three decades.

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Disclosures

The views and opinions expressed herein are those of the authors and do not reflect the views of Goldman Sachs. The multi-factor model seeks to identify certain shortcomings in current asset allocation tools. Investors must also consider suitability, liquidity needs, and investment objectives when determining appropriate asset allocation.